Краткие сообщения ОИЯИ №5 [31]-88 JINR Rapid Communications No.5 [31]-88
УДК 539.12.01

PHENOMENOLOGY OF PRODUCTION PROCESSES OF AN ASYMPTOTICALLY LARGE NUMBER OF HADRONS

I.D.Mandzhavidze, A.N.Sissakian

A classification is proposed for possible asymptotic production cross sections σ_n with respect to n which is independent of concrete models of strong interactions and a physical meaning of the classification is explained on the basis of the statistical physics picture.

The investigation has been performed at the Laboratory of Theoretical Physics, JINR.

Феноменология процессов рождения асимптотически большого числа адронов

И.Д.Манджавидзе, А.Н.Сисакян

Предлагается классификация возможных асимптотических сечений рождения n адронов σ_n, не зависящая от конкретных моделей сильных взаимодействий, и на основе статфизической картины поясняется физический смысл рассматриваемой классификации.

Работа выполнена в Лаборатории теоретической физики ОИЯИ.

1. Let us describe the multiple production of hadrons when their number n is very large

$$n \gg \bar{n}(s). \tag{1}$$

Here $\bar{n}(s)$ is the mean multiplicity defining the natural scale of values n at a given energy. Interest in this region (1) stems from the expectation to get further information that would refine our knowledge of strong interactions.

Since there is no quantitative theory, it would be well to develop a general picture of physical phenomena in the region (1) which is independent of model notions formed by investigations in the region $n \sim \bar{n}$. We shall construct this phenomenological picture on the basis of the statistical mechanics by representing a final state of the process as a (microcanonical) ensemble . For this purpose we introduce the density matrix $\rho(\beta, z)$, such that the production cross section of n particles is

$$\sigma_n(s) = \frac{1}{2\pi i} \oint \frac{dz}{z^{n+1}} \frac{1}{2\pi i} \frac{1}{\sqrt{s}} \int_{Re\beta>0} \frac{d\beta}{2\pi} \beta^2 I_1(\beta\sqrt{s})\,\rho, \tag{2}$$

where I_1 is the Bessel function of an imaginary argument (for a detailed derivation of formula (2) see, for instance, ref.[1]).

At large n integration in (2) can be performed by the saddle point method. First, we should find the solutions of the equations (state)

$$\sqrt{s} = \frac{\partial}{\partial \beta} \ln \rho(\beta, z), \tag{3}$$

$$n = z \frac{\partial}{\partial z} \ln \rho(\beta, z). \tag{4}$$

Under this definition of integrals in (2), $1/\beta$ means the gas temperature of particle production and z means activity (i.e., $(1/\beta)\ln z$ is the chemical potential).

Now we take advantage of the fact that the asymptotics σ_n with respect to n is defined by the leftmost singularity $\rho(\beta, z)$ in z and weakly depends on the nature of the singularity. On the basis of the statistical mechanics we assume $\rho(\beta, z)$ to be a regular function of z inside the circle $|z| = 1$[2]. If z_c is the leftmost singularity, then from general considerations one would expect one the following possibilities to be realised:

$$\begin{aligned} &a)\ z_c = 1 \\ &b)\ z_c = \infty \\ &c)\ 1 < z_c < \infty \end{aligned} \tag{5}$$

thus providing a classification of possible asymptotics we search for. Now let us elucidate what physical conditions the quantity z_c depends on.

2. First, it is to be noted that the singularity $\rho(\beta, z)$ at finite z is treated as an indication of a phase transition[2,3]. For instance, let β be such that particles are combined into clusters*. Then, the number of clusters of ℓ particles is $\sim e^{-\beta\sigma\ell^{d-1/d}}$, where $\sigma\ell^{d-1/d}$ is the cluster surface energy ($\ell^{d-1/d}$ is the cluster surface area). Then

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* *More precisely, the decay of clusters produces particles.*

$$\rho(\beta, z) \sim \exp\{ \sum_{\ell=1}^{\infty} z^{\ell} e^{-\beta\sigma\ell^{d-1/d}} \tag{6}$$

will be singular at $z = 1$. This singularity indicates a first order phase transition (condensation).

In calculating the relevant σ_n we consider the following analog model. Let us cover the volume, into which particles are produced, by the net and let the presence of a particle in the node be denoted by (-1) and the absence by (+ 1).

Now we take advantage of the fact that this model of lattice gas is well described by the Ising model. Switching on a magnetic field $\mathcal{H}$ we can contral the number of down spins, i.e., the number of produced particles. This means that activity $z = e^{-2\beta\mathcal{H}}$ and $\mathcal{H}$ acts as a chemical potential.

Then, the density matrix ρ in the continuous limit is determined by the functional integral[4]

$$\rho(\beta, z) = \int \mathfrak{D}\mu\, e^{-S_{\lambda}(\mu)}, \tag{7}$$

where the action

$$S_{\lambda}(\mu) = \int d\mathbf{x}[\frac{1}{2}(\nabla\mu)^2 - \epsilon\mu^2 + d\mu^2 - \lambda\mu] \tag{8}$$

and

$$\epsilon \sim (1 - \beta_c/\beta); \quad \lambda \sim \mathcal{H}. \tag{9}$$

Here $1/\beta_c$ is the phase transition temperature. Assume that $\beta > \beta_c$, i.e., the average spin $<\mu> \neq 0$. To simplify the calculations we assume that $\beta/\beta_c >> 1$ (this ensures small fluctuations in the vicinity of the chosen $<\mu>$).

Singularity in $\mathcal{H}$ arises due to the following reason. At $\mathcal{H} = 0$ the potential

$$V = -\epsilon\mu^2 + d\mu^2; \quad \epsilon, a > 0 \tag{10}$$

has two minima at $\mu_{\pm} = \pm\sqrt{(\epsilon/2a)}$. Switching on $\mathcal{H} < 0$ we destroy the degeneracy. The left minimum at $\mu = -\sqrt{(\epsilon/2a)}$ appears to be lower than the right one. Then, the system in the right minimum (it is described by up spins, which means the absence of produced particles) turns out to be unstable: a tunneling into a lower (stable) minimum is possible.

The above instability is associated with the branching point in the complex plane $\mathcal{H}$ at $\mathcal{H} = 0$ and the discontinuity provides [6]

$$\mathrm{Jm}\,\rho(\beta, z) \simeq \frac{a_1(\beta)}{\mathcal{H}^2} e^{-\frac{a_2(\beta)}{\mathcal{H}^2}}, \tag{11}$$

where a_1 and a_2 are independent of Ω.

Using (11) we find that the solution (4) has the form

$$\bar{z} \simeq \exp\left\{\frac{8\beta^2 a_2}{n}\right\}^{1/3}, \tag{12}$$

which corresponds to the following asymptotics

$$\rho_n(\beta) \propto e^{-3(\beta^2 a_2)^{1/3} n^{2/3}}, \tag{13}$$

i.e., we see that the singularity at $z = 1$ is associated with the following class of asymptotics: $\sigma_n > 0(e^{-n})$.

It is to be noted that $\rho_n(\beta)$ is determined by the contribution of only $\mathrm{Jm}\,\rho$ and metastable states, whose decay is described by $\mathrm{Re}\,\rho$, are insignificant.

The contribution considered above describes the decay of an unstable (with respect to particle production) state. This decay produces clusters and if the size of a cluster is larger than a critical one, cluster's size infinitely increases with time. During this motion the cluster walls "accelerate", i.e., the larger the number of particles forming a cluster, the smaller energy is needed to add one particle into a cluster[5,6]. Just this phenomenon is observed in the decrease of $\bar{z}$ with increasing n, see (12).

3. Let us continue the discussion of (7) at $\beta < \beta_c$. In this case the potential (10) has the only minimum at $z = 0$. By switching on the external field there arises a mean field $\bar{\mu} = \bar{\mu}(\mathcal{H})$ that in the first apptoximation can be found from

$$z|\epsilon|\mu + 4a\mu^3 = \lambda. \tag{14}$$

At large $\mathcal{H}$, which corresponds to asymptotics in n, eq. (14) has the solution

$$\bar{\mu} \simeq (\lambda/4a)^{1/3}. \tag{15}$$

Estimation of the integral (7) in the vicinity of this minimum provides

$$\rho(\beta, z) \sim e^{\gamma(\ln z)^{4/3}} \quad ; \quad \gamma = \gamma(\beta) > 0. \tag{16}$$

We see that in the case under consideration the singularity is at $z = z_c = \infty$.

Equation (4) has the solution

$$\bar{z} \sim \exp\left(\frac{3n}{4\gamma}\right)^3 \tag{17}$$

that increases (in contrast with the one considered in sec. 2) with n. With (17) one can easily find that

$$\rho_n(\beta) \sim e^{-\bar{\gamma} n^4}, \quad \bar{\gamma} = \bar{\gamma}(\beta) > 0, \tag{18}$$

i.e., decreases faster than e^{-n}.

4. We should like to emphasize that the above considered analog model does not account for the nature of the singularity $\rho(\beta, z)$ at finite z. Therefore, we should clarify our arguments. However, it follows from general considerations that the singularity $\rho(\beta, z)$ at finite z testifies to a phenomenon similar to the phase transition. This means that the particpe production should be considered as a result of the decay of "clusters". This process can be described by refining formula (6) as follows: Let the probability of tne i th "cluster" (i = 1,2,...) of mass m_i to decay into n_i particles be $\omega_{n_i}(m_i)$. Then, neglecting the interaction between "clusters" (see also ref. /7/) we have

$$\rho(\beta, z) = \exp\left\{ \int_{m_0}^{\infty} \frac{dm}{m} \sigma(m)\, e^{-\beta m}\, t(z, m) \right\}, \tag{19}$$

where $\sigma(m)$ is proportional to the average number of mass clusters and

$$t(z, m) \equiv \sum_{n=1}^{\infty} z^n \omega_n(m), \quad t(1, m) = 1. \tag{20}$$

The "Boltzmann" factor $e^{-\beta m}$ in (19) arises due to the energy-momentum conservation laws. Assuming in (19) $(m - (1/\beta)\ln t)$ to be the total energy of a "cluster" and replacing the integral by the sum, we can arrive at a formula analogous to (6). The phase transition, described in sec. 2, in terms of formula (19) corresponds to the integral divergence in the upper limit at $z = 1$.

As an example, let us consider the case when $t(z, m)$ is singular at $z = z_c$, $1 < z_c < \infty$. For instance, let

$$t(z,m) = \left(\frac{z_c - 1}{z_c - z}\right)^{\nu}, \qquad \nu > 0. \tag{21}$$

Taking into account that an average number of particles produced in the decay of a "cluster" of mass m

$$\bar{n}(m) = \frac{\partial}{\partial z} \ln t(z, m)\,|_{z=1} \tag{22}$$

we can express z_c through $\bar{n}(m)$. For formula (21) we get that

$$z_c(m) = 1 + \frac{\nu}{\bar{n}(m)}. \tag{23}$$

It is to be noted that irrespective of the type of singularity only the assumption about $t(z,m)$ tending to infinity at $z = z_c$ defines by (22) the position of a singularity on the right from unity. Moreover, with increasing m the singulatiry moves to the left. Then, according to the momentum energy conservation laws the production of a particle in the decay of one "cluster" will dominate in the asymptotics in n. Indeed, the production of particles in the decay of two "clusters" $\sim t^2(z, s/4)$ and this contribution in the z plane are associated with the singularity

$$z_c^{(2)} = 1 + \frac{\nu}{\bar{n}(s/4)} > z_c^{(1)} = 1 + \frac{\nu}{\bar{n}(s)}.$$

Assuming that correlations between particles produced in the decay of one "cluster" differ from those between particles produced in the decay of various "clusters" the afore-said implies the presence of a "phase transition" which is reflected in the change of the nature of correlations with increasing n. However, this transition is smooth without sharp changes. Therefore, it is better called the "structure phase transition".

Thus, after a structure phase transition

$$\rho(\beta, z) \simeq \int \frac{dm}{m} \sigma(m)\, e^{-\beta m}\, t(z, m). \tag{24}$$

Hence, one can easily see that

$$\sigma_n(s) \propto e^{-\nu^n/\bar{n}(s)} \quad , \tag{25}$$

i.e., corresponds to the KNO scaling.

Using the above mechanism of phase transition, one can easily find the range of values of n, where the estimate (25) is valid. The correction to (25) due to the production of two clusters is $\sim \exp(-\nu^n/\bar{n}(1/4))$. Hence, if

$$n \ge \frac{1}{\nu} \frac{\bar{n}(s)\,\bar{n}(s/4)}{\bar{n}(s) - \bar{n}(s/4)} , \tag{26}$$

the estimate (25) is valid. Assuming that the differences $\bar{n}(s) - \bar{n}(s/4) \sim 1$ the structure phase transition begins at $n \sim \bar{n}^2(s)$ (if the production of two clusters has no additional smallness).

Using (21) one can find the ratio of dispersion D to $\bar{n}$ with regard to the production of two clusters

$$\frac{D^2(s)}{\bar{n}^2(s)} \sim 1 - a\frac{\bar{n}(s/4)}{\bar{n}(s)} , \tag{27}$$

where the positive constant a takes into account a relative weight of the production of two clusters. We see that the ratio of dispersion to average multiplicity increases with energy.

5. Now we shall formulate the main results of the paper.

a) According to our classification the asymptotics

$$\sigma_n > 0(e^{-n}) \tag{28}$$

is associated with the first order phase transition (condensation). Since the ground state of the effective potential of the interaction of hadrons is hardly expected to be unstable, the asymptotics (28) is improbable; one would expect that

$$\sigma_n \le 0(e^{-n}) . \tag{29}$$

b) The asymptotics

$$\sigma_n = 0(e^{-n}) \tag{30}$$

of necessity has the form of the KNO scaling. A similar asymptotics is predicted by the inverse binomial distribution /8/, in the distribution[8] in the distribution over multiplicity of partons in the QCD jet, in the cascade processes[9] and provides the best agreement with experiment[10]. A structure phase transition at $n \sim \bar{n}^2(s)$ is typical of the asymptotics (30), which naturally accounts for the observed violation of the KNO scaling and increase in $(D/\bar{n})$ with energy[11].

c) For the asymptotics

$$\sigma_n < 0(e^{-n}), \tag{31}$$

which is typical of the multiperipheral modes[12], a slight violation (at least at modern accelerator energies) of the KNO scaling appears to be a pure chance since the scale-invariant structure is the privilege of phase transitions.

In conclusion, we should like to emphasize that the asymptotics (30) and (31) have roots in various physical phenomena and only the experimental information in the region $n \gg \bar{n}$ (up to $n \sim \bar{n}^2$) may elucidate which of them is realized in practice.

Acknowlegement

The authors are grateful to Professors V.G.Kadyshevsky, V.A.Matveev, L.A.Slepchenko and A.N.Tavkhelidze for interest in this work and discussions.

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Received on July 5, 1988.